Quality Improvement of Finite Element Mesh Models Modified by Mesh Deformation

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Abstract:
In product shape design, CAD model modification, meshing of the modified CAD models and CAE analysis are repeated until a desired product shape is obtained. For reducing the number of meshing processes which are unstable and time-consuming, many methods for directly modifying mesh models have been proposed. However, the deformed mesh models often include many bad elements, such as slivers, and lose the mesh density and shape approximation accuracy of initial mesh models. To solve this problem, in this paper, we propose a quality improvement method for the deformed tetrahedral mesh models. Our method, based on Optimal Delaunay Triangulation smoothing, improves element shape quality by optimizing the position and number of vertices on the sharp edges, on the surfaces, and in the volumes, in that order. In our method, mesh density of the initial mesh model is preserved in the deformed mesh models using the density fields represented by regular grids. The shape approximation accuracy is also kept in the resulting mesh models, depending on the shape tolerances extracted from the initial mesh model. We demonstrate effectiveness of our method by application to some mesh models.

Keywords: Quality Improvement, Tetrahedral Mesh, Mesh Deformation, Optimal Delaunay Triangulation Smoothing

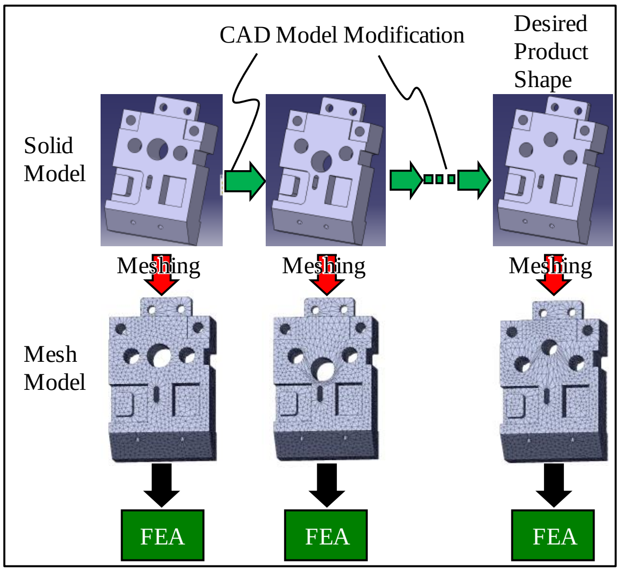
1. Introduction

Mesh deformation is used in CG character design, animations, and product shape design. In particular, mesh deformation plays an important role for realizing efficient product shape design. In present product shape design, to find optimal shapes, product shapes are modified by using solid models in CAD system. Then, mesh models are generated through meshing of the solid models. And the product shapes are evaluated by Finite Element Analysis (FEA) using the mesh models. These processes (CAD model modification, meshing, and FEA) are repeated until a desired product shape is obtained (Fig. 1(a)). However, meshing is unstable and time-consuming. Therefore, modifying the product shapes using mesh models can realize an efficient product shape design process because it can cut down on the number of times of meshing (Fig. 1(b)).

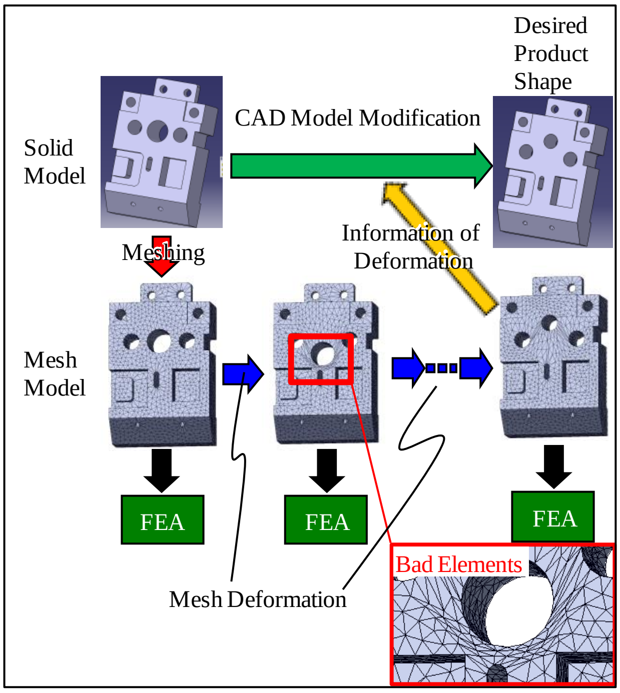
Many mesh deformation methods for modifying product shapes have been proposed [1-6]. However, the resulting deformed mesh models often include many bad (distorted) elements, and lose mesh density and shape approximation accuracy of the initial mesh models. The quality of element shape of deformed mesh models must be improved because bad elements cause low accuracy and inefficiency of FEA. In addition, deformed mesh models should have the mesh density and approximation accuracy of the initial mesh models because changes of mesh density and shape approximation accuracy influence the accuracy of FEA and desired results may not be provided.

In this paper, we propose a quality improvement method for deformed mesh models. Our method is based on Optimal Delaunay Triangulation (ODT) smoothing [7] and improves quality of element shape by optimizing the positions and the number of vertices on the sharp edges, on the surfaces, and in the volumes, in that order. In our method, the regular grid representing target mesh density fields is generated and used to preserve mesh density of the initial mesh models. In addition, the shape tolerances are extracted from the initial mesh models to keep the shape approximation accuracy of initial mesh models in the resulting mesh models. Therefore, our method can provide high quality deformed mesh models with mesh density and approximation accuracy of initial mesh models. In this paper, we handle tetrahedral mesh models and assume that the models consist only of planar or cylindrical surfaces.

The rest of this paper is organized as follows. First, related works are shown in Section 2. In Section 3, the outline of our method is described. Then, derivations of the control values to preserve mesh density and shape approximation accuracy of initial mesh models are explained in Section 4. In Section 5, a method of quality improvement of deformed mesh models by using phased ODT smoothing and insertion and removal of vertices is proposed. In Section 6, we demonstrate effectiveness of our method through application of the method to some deformed mesh models. Finally, we conclude and discuss our future work in Section 7.



(a) Present process of product shape design



(b) Efficient process of product shape design using mesh deformation & bad elements

Figure 1 Effectiveness of mesh deformation in product shape design

2. Related Works

2.1. Mesh Deformation Method

There are many mesh deformation methods to realize efficient product shape design [1-6]. Takano et al. [1] proposed a tetrahedral mesh deformation method based on space embedding. Their method can change the three types of dimensions: distance between two parallel planes (DP), radius of the cylinder (RC), and position of the local object on the plane (PO) of the tetrahedral mesh models.

Onodera et al. [2] proposed a parametric morphing method for tetrahedral mesh models based on surface fitting and Laplacian smoothing. Their method can change DP, RC, PO, and radius of sphere for the mesh models.

Ogawa et al. [3] proposed a 3D triangular mesh deformation method using some constraints. Their method can interactively and intuitively deform the 3D triangular assembly mesh models including some non-manifold parts. In addition, the method can deform tetrahedral mesh models by combining with their related work [4].

Xian et al. [5] proposed 3D triangular mesh editing method using some cages. Their method can deform mesh models while preserving rectangular parallelepipeds and cylinders. The method also can handle assembly models and can apply to large mesh models.

Sawai et al. [6] proposed tetrahedral mesh deformation method using CAD data. Their method can deform mesh models by specifying parameters of the shape features. In addition, their method can deform loft form, and modified solid models can easily be generated from resulting mesh models through the relation between them.

Because their deformation is done only by moving the vertices, mesh models after large deformations have a lot of distorted elements and lose the mesh density and the shape approximation accuracy of initial mesh models. Therefore, deformed mesh models must be improved in order to use them in FEA.

2.2. Quality Improvement Method

Many methods of quality improvement of mesh models are proposed [7-9]. In particular, the methods based on Optimal Delaunay Triangulation (ODT) and Centroidal Voronoi Tessellation (CVT) [7-9] can provide high-quality mesh models.

The methods based on ODT (ODT smoothing) [7] improve quality of mesh models based on the certain mesh energy. The improvement is done by iterative repositioning of vertices and edge (face) flipping. Repositioning is done by replacing vertices to the average of circumcenters of its neighboring elements. Therefore, the computational cost is very low.

In general, ODT smoothing fixes boundary vertices and moves interior vertices. Therefore, it cannot improve the elements near the boundary of mesh models. Tournois et al. [8] proposed Natural Optimal Delaunay Triangulation (NODT). NODT can improve the quality of all element shapes without deforming the shape of mesh models.

The methods based on CVT [9] improve quality of mesh models by iterative generation of Voronoi Diagram and Delaunay Triangulation (Tetrahedration) using the centroid of the Voronoi regions. In general, ODT smoothing converges faster than the CVT-based method.

Element shape qualities, mesh density, and approximation accuracy of the initial mesh model should be preserved after mesh deformation. Because ODT smoothing and the CVT-based methods improve qualities of element shapes of mesh models only by moving vertices, they cannot recover mesh density and shape approximation accuracy of the initial (before deformation) mesh model. Therefore we combine ODT smoothing with insertion and removal of vertices to improve element shape qualities of deformed mesh models and to preserve mesh density and approximation accuracy of the initial mesh model.

In original ODT smoothing, the vertices on the boundary of the mesh model are fixed, therefore it cannot improve the qualities of the boundary element shapes of mesh models well. Tournois's method [8] solved this problem and proposed an extension method for improving qualities of the boundary element shapes of mesh models in the framework of the ODT (called NODT). In our method, in order to improve the boundary elements (e.g. surface triangles for the tetarahedral mesh), the ODT is applied first to sharp edges, then to the surface and finally to the inner volume in sequence while fixing each boundary. This strategy will provide better surface mesh quality because the ODT energy for the surface is minimized independent of the volume. In addition, our method is applicable to 3D surface mesh models.

3. Outline of Quality Improvement of Deformed Mesh Models

The outline of our method is shown in Fig. 2. First, mesh density, geometric information (sharp edges, surface regions, and surface parameters), and shape tolerances of a given initial mesh model are extracted (A-1). Secondly, the mesh model is deformed by a mesh deformation method [1][2] (A-2). After deformation, a grid of target mesh density field is generated by using the mesh density of the initial mesh model (A-3). In this step, geometric information of the deformed mesh model is also extracted. Finally, the positions and the number of the vertices are optimized on the bases of ODT, the grid of target density field, and shape tolerances of the initial mesh model (A-4).

To improve quality of all elements, our method improves quality of elements on the sharp edge (mesh edges), on the surface (surface triangles), and in the mesh model (tetrahedra), in that order. ODT smoothing, insertion of vertices, and removal of vertices are iteratively applied to deformed mesh models in mesh quality improvement step. In our method, the insertion of a vertex is done by edge split, and the removal of a vertex is done by edge collapse in order to keep the topological consistency of the mesh model. In the following sections, we first outline a method for extracting the density and approximation accuracy of the input mesh model. Then we explain ODT smoothing and show a way of applying it to the sharp edges, the surface meshes, and the inner mesh. Finally, we describe a method for the insertion and removal of vertices.

4. Derivation of Control Value (A-1, A-3)

4.1. Derivation of Target Mesh Density Field

In order to preserve the mesh density of the initial mesh model in the deformed mesh models, the density information of the initial mesh model is extracted, and the target mesh density field is represented by using a regular grid. First, at each vertex in the initial mesh model, the mesh density is calculated as the average of reciprocals of its incident edge lengths. Secondly, the mesh models are deformed by a mesh deformation method [1] [2]. Then, a regular grid covering the deformed mesh model is generated, and the mesh density is given to each cell of the grid. In order to give the mesh density to cells of the grid, the mesh density $\rho(g)$ of each grid point g is calculated by Eq.(1) (the interpolation by barycentric coordinates):

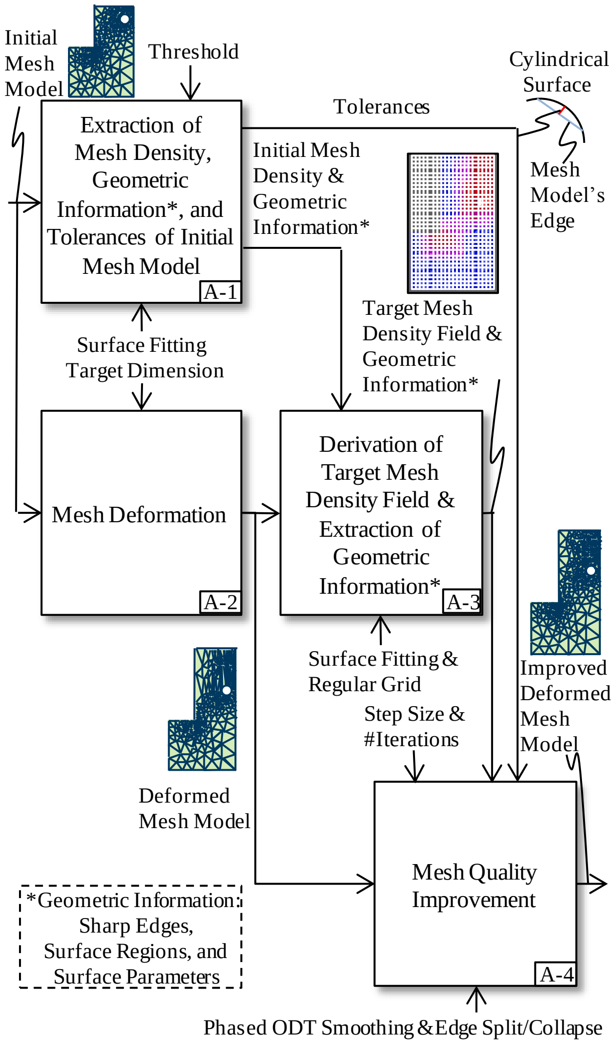


Figure 2 Outline of our method

$$\rho(g) := \frac{1}{|\tau_j|}\sum_{i\in\tau_j}|\tau_j^i|\rho_i. \tag{1}$$

In Eq. (1), τ_j is a tetrahedron which includes g, $|\tau_j|$ the volume of τ_j, τ_j^i a tetrahedron where the vertex $i(\in\tau_j)$ is replaced by g, and ρ_i mesh density of i. Finally, the mesh density of each cell is given as the average of the nonzero mesh densities of its grid points.

4.2. Extraction of Sharp Edges, Surface Regions, Surface Parameters, and Approximation Error

To keep shape approximation accuracy after deformation, geometric information (sharp edges, surface regions, surface parameters,) and shape tolerances of the initial mesh model are extracted (Fig. 2 A-1). Geometric information is extracted as follows. First, the sharp mesh edges are found by thresholding of dihedral angles. Secondly, corner points having three or more sharp mesh edges are identified. Then, the connected sequence of sharp mesh edges between two corner points or that forming a loop is identified as a sharp edge, and a set of triangles surrounded by sharp edges is identified as a surface region. By surface fitting, each region is classified as either a planar or cylindrical region, and surface parameters are extracted. Finally, each sharp edge is classified as either a straight line, a circular arc, a circle, or other, according to types of regions sharing it.

At each mesh edge in the cylindrical regions, the error of shape approximation is defined as the difference between distance from its midpoint to axis of the fitted cylinder and radius of the cylinder region. The shape tolerance of each cylindrical region is defined as the maximum error of the shape approximation of the edges in that region in the initial mesh model.

After deformation, at first, sharp edges and surface regions of the deformed mesh model are extracted using result of the extraction of the initial mesh model (Fig. 2 A-3). Then surface parameters are recalculated in order to preserve shape of the deformed mesh model in the mesh quality improvement.

5. Mesh Quality Improvement (A-4)

5.1. Outline of Mesh Quality Improvement

In mesh quality improvement, ODT smoothing is combined with insertion and removal of vertices (Fig. 3). First, the quality of element shapes of the deformed mesh model is improved by ODT smoothing. However the deformed mesh model loses shape approximation accuracy of the initial mesh model. Therefore, insertion of vertices are applied to the deformed mesh model using tolerances from the initial mesh model. Then, because the mesh density of the deformed mesh models differs from that of initial mesh model, insertion and removal of vertices are applied to the deformed mesh model using the grid of the target density field. In our method, these three operations are iterated until all edges of the deformed mesh model satisfy shape approximation accuracy and target mesh densities.

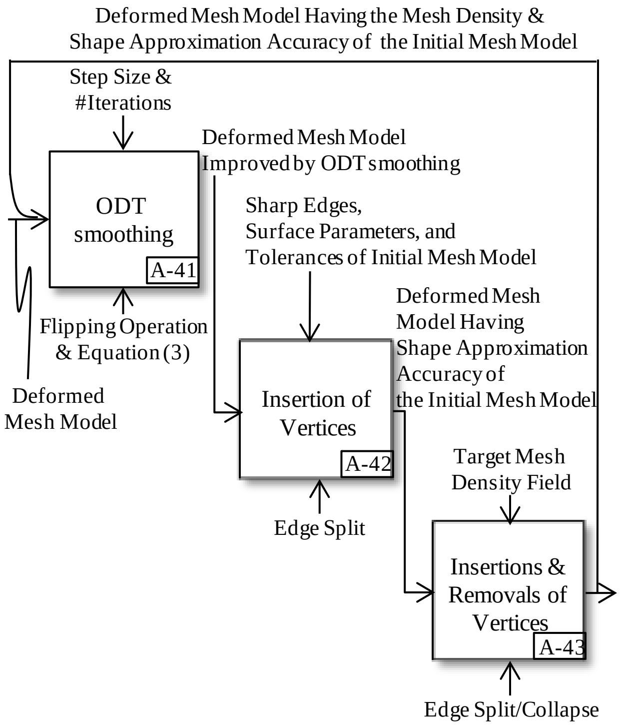


Figure 3 Outline of mesh quality improvement

5.2. ODT Smoothing[7]

ODT smoothing is one of mesh improvement methods, which minimize an error function, Eq. (2):

$$E(T) = \int_{\Omega} \left|u_{T,I}(\boldsymbol{x}) - u(\boldsymbol{x})\right| \rho(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \,. \tag{2}$$

In Eq. (2), $u(\boldsymbol{x})$ is $\|\boldsymbol{x}\|^2$, ρ a given density function defined on a convex domain $\Omega \subset \mathbb{R}^n$, T a simplicial subdivision (mesh) of Ω, and $u_{T,I}(\boldsymbol{x})$ the piecewise linear approximation of $u(\boldsymbol{x})$ based on T. In ODT smoothing, repositioning of vertices and flipping operation are iterated in order to minimize Eq.(2) (Fig. 4). In repositioning, new positions of vertices are calculated using Eq.(3):

$$\boldsymbol{x'}_i = (1-\alpha)\boldsymbol{x}_i + \frac{\alpha}{|\omega_i|} \sum_{\tau_j \in \omega_i} \boldsymbol{c}_j \,. \tag{3}$$

In Eq. (3), $\boldsymbol{x}_i$ is the position of vertex i before repositioning, $\boldsymbol{x'}_i$ new position of vertex i after repositioning, ω_i a set of elements which includes i, $\boldsymbol{c}_j$ position of circumcenter of element τ_j, and α is step size $(0 < \alpha \leq 1.0)$. The flipping operation is applied so that all elements satisfy Delaunay condition.

ODT smoothing can provide high quality mesh models at low computational cost. However, in general, it cannot improve qualities of boundary element shapes. In order to improve them, we propose phased ODT smoothing. In phased ODT smoothing, ODT smoothing is applied in three phases: sharp mesh edge improvement, surface triangle improvement, and tetrahedron improvement. In the following section, we explain each

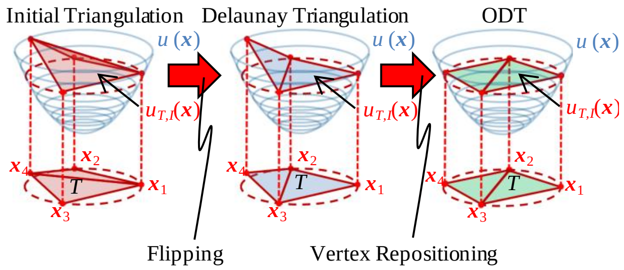


Figure 4 Improvement step of ODT smoothing

phase.

5.3. Sharp Mesh Edge Improvement

For sharp mesh edges on a straight sharp edge, the repositioning of vertices by ODT smoothing is done by using Eq. (3), where midpoints of edges are used as $\boldsymbol{c}_j$ (stated exactly, this smoothing is called CPT smoothing [7]). On the other hand, circle (or circular arc) sharp edges shrink when their vertices are moved by Eq. (3). Therefore, as shown Fig. 5, each vertex position on the circle (or circular arc) sharp edge is parameterized by its central angle θ, and the new central angle is calculated by Eq. (3). A new vertex position is determined as a position on the circular arc or the circle corresponding to the new central angle. In this phase, there are no flipping operations.

5.4. Surface Triangle Improvement

For triangles on each planar surface region, ODT smoothing is done by Eq. (3) where circumcenters of surface triangles are used as $\boldsymbol{c}_j$. In order to avoid the generation of distorted triangles near the region boundary, the barycenter is used as $\boldsymbol{c}_j$ in Eq. (3) for triangles that have at least one vertex on the region boundary. While elements of planar regions are improved, cylindrical regions shrink by the ODT smoothing. Therefore in our method, the new vertex position $\boldsymbol{x}_i^{new}$ of $\boldsymbol{x}_i$ is calculated as follows (Fig. 6).

Step 1) Circumcenters (or barycenters) $\{\boldsymbol{c}_j\}$ of neighboring triangles of vertex i are projected onto cylindrical surfaces.

Step 2) The projected $\{\boldsymbol{c}_j\}$ and $\boldsymbol{x}_i$ are mapped onto a $r\theta$-z plane that is a development of the cylinder. $\{\hat{\boldsymbol{c}}_j\}$ and $\hat{\boldsymbol{x}}_i$ denote mapped $\{\boldsymbol{c}_j\}$ and $\boldsymbol{x}_i$.

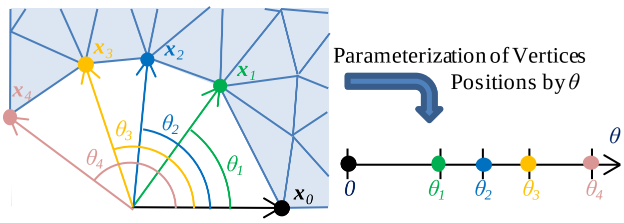


Figure 5 Parameterizing vertices positions for ODT smoothing on the circle (or circular arc) sharp edge

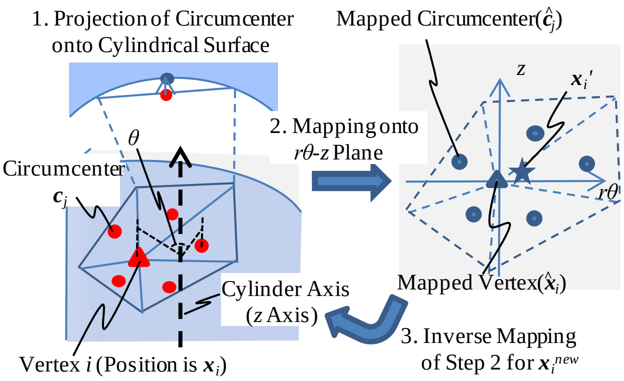


Figure 6 ODT smoothing for cylindrical surface

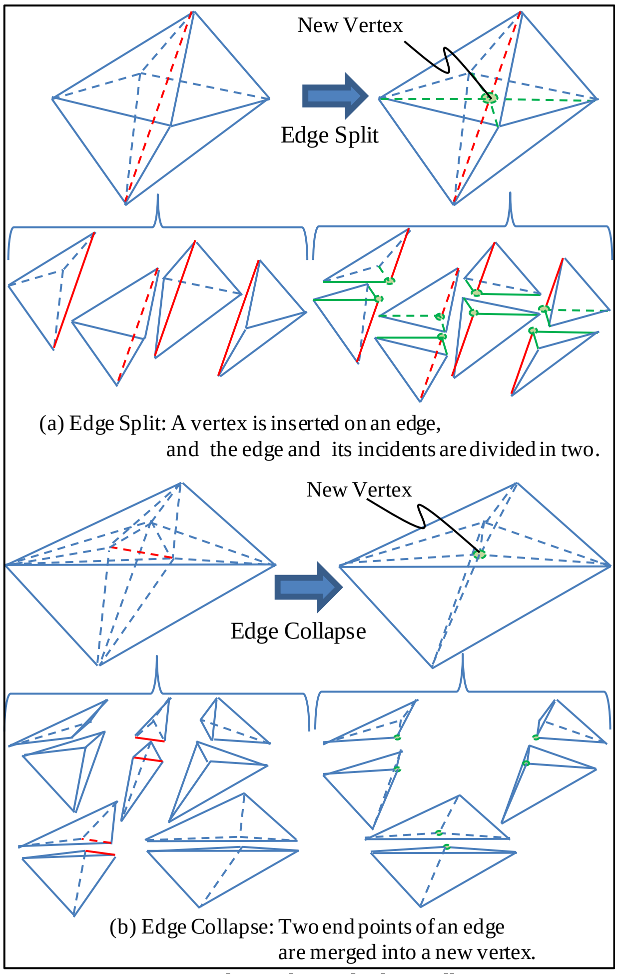


Figure 7 Edge split and edge collapse

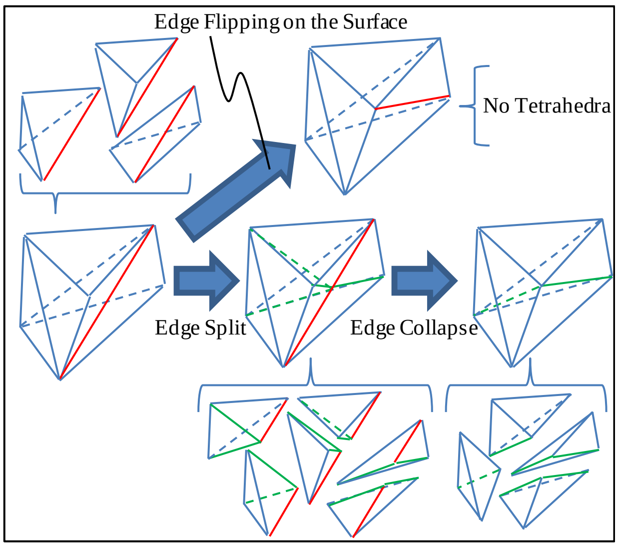


Figure 8 Proposed method for edge flipping

Step 3) The $\boldsymbol{x}'_i$ is calculated using $\{\hat{\boldsymbol{c}}_j\}$ and $\hat{\boldsymbol{x}}_i$.

Step 4) $\boldsymbol{x}_i^{new}$ is derived from $\boldsymbol{x}'_i$ by inverse mapping of Step 2.

On the surface, ODT smoothing includes edge flipping. Flipping operation of the surface edge must be done considering interior elements. Because there are connections between surface vertices and interior vertices, not all edges can be flipped by edge flipping on the surface. In our method, the edges are flipped by using edge split and edge collapse (Fig. 7, 8).

In the edge flipping, as shown Fig. 8, the edge is split at first, then a new edge whose two vertices are on the surface is collapsed. If slivers or inverted elements would occur during the edge flipping, flipping is not applied to that edge. Because some edges cannot be flipped, some triangles do not satisfy the Delaunay condition. If circumcenters are used as $\boldsymbol{c}_j$ in the Eq. (3) for such triangles, element shape qualities become worse near the triangles. Therefore in our method, the barycenters instead of their circumcenters are used as $\boldsymbol{c}_j$ in the Eq. (3) for triangles which do not satisfy the Delaunay condition.

5.5. Tetrahedron Improvement

For tetrahedra, interior vertices of the model are moved by using Eq. (3) where circumcenters of tetrahedra are used as $\boldsymbol{c}_j$. In order to avoid the generation of distorted tetrahedra near the surface, the barycenter is used as $\boldsymbol{c}_j$ in Eq. (3) for the tetrahedra that have at least one vertex on the surface. In addition, the barycenter is used for tetrahedra which do not satisfy the Delaunay condition in Eq.(3).

In the flipping operation, *Flipping 2-3*, *Flipping 3-2*, and *Flipping 4-4* [9] are used (Fig. 9). In Flipping 2-3, for two neighboring tetrahedra sharing a triangle, an edge connecting the opposite vertices is generated, and the shared triangle is removed. Flipping 3-2 is an inverse operation of Flipping 2-3. In Flipping 4-4, for an octahedron which consists of four tetrahedra sharing one edge, the edge is swapped.

The triangles or edges which satisfy the following three conditions are flipped.

C1) the triangle or edge has neighboring tetrahedra which do not satisfy the Delaunay condition.

C2) After flipping, all resulting tetrahedra satisfy the Delaunay condition.

C3) Minimum quality of the resulting tetrahedra after flipping is higher than before flipping.

5.6. Insertion and Removal of Vertices

In order to preserve the mesh density and shape approximation accuracy of the initial mesh model in the deformed mesh models, vertices are inserted or removed. Edge split is used for inserting vertices (Fig. 7(a)) and edge collapse is used for removing vertices (Fig. 7(b)).

In order to preserve the shape approximation accuracy of the initial mesh model, at first, surface mesh edges which have a larger approximation error than the tolerance of the cylindrical surface are found. Then the vertices are inserted at midpoints of the edges by using edge split. Finally, the vertices generated by edge split are moved onto the cylinder. Thus the shape approximation accuracy of the deformed mesh becomes similar to one of the initial mesh model.

To preserve mesh density of the initial mesh model, at first, for each edge e in the deformed mesh model, the mesh density ρ_e is calculated as the inverse of its length.

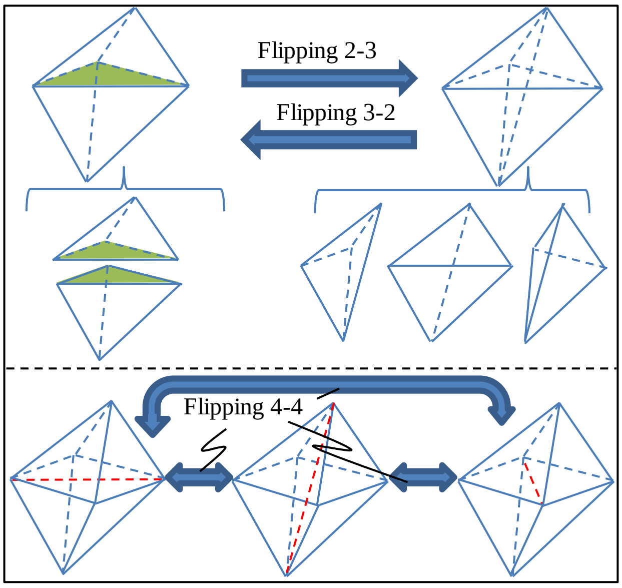


Figure 9 Flipping operation in the tetrahedron improvement phase

Table 1 Position of the new vertex after edge collapse

Target Edge	Position of The New Vertex
Surface Edge Connecting One Corner Point	the Corner Point
Surface Edge Connecting One Boundary Point*1	the Boundary Point
Other Surface Edge on a Cylindrical Region	Position on the Cylindrical Surface Coresponding to Midpoint of the Edge
Interior Edge Neighboring One Surface Point*2	the Surface Point
Other Edge	Midpoint of the Edge

*1 A vertex on a sharp edge

*2 A vertex on a surface

Secondly, each mesh density ρ_e is compared with the mesh density $\hat{\rho}_e$ given by the grid cell of the target mesh density field, which includes the midpoint of the edge e. If the mesh density ρ_e is smaller than $\gamma\hat{\rho}_e$, the edge e is split. On the other hand, if the mesh density ρ_e is larger than $\delta\hat{\rho}_e$, the edge e is collapsed. γ and δ are constants and called density thresholds in this paper. After edge split, the new vertex is moved onto the cylinder if the edge e is on the cylindrical surface. In order to preserve model shape, edge collapse is not applied to following edges.

1. Edge connecting two corner points.
2. Non-sharp mesh edge connecting two vertices on a sharp edge.
3. Interior edge connecting two vertices on the surface

The position of the new vertex after edge collapse is decided under the rule shown in Table 1.

6. Results and Evaluations

6.1. Definition of Quality Evaluation Measure, Mesh Deformation Method, and Density Thresholds

In our research, stretch is used as a quality measure of element shapes. Stretch is 1 for a regular tetrahedron. The value of stretch becomes smaller for distorted tetrahedron and it is 0 for degenerated tetrahedron (like a sliver). The stretch $st(\tau)$ of a tetrahedron τ is defined by Eq. (4):

$$st(\tau) = \frac{6\sqrt{6}\,V(\tau)}{(\max_{e \in E_\tau} l(e))S(\tau)} \quad . \qquad (4)$$

In Eq. (4), $V(\tau)$ is the volume of τ, $S(\tau)$ the surface area of τ, E_τ a set of edges of τ, and $l(e)$ is the length of edge e.

At each edge, mesh density error $\varepsilon_\rho(e)$ is calculated by Eq. (5):

$$\varepsilon_\rho(e) = \frac{|\rho_e - \hat{\rho}_e|}{\rho_e}. \qquad (5)$$

The error is used to evaluate mesh density of each mesh model.

In our experiments, we used Takano's method [1] for mesh deformation, and the density thresholds γ and δ were set to 2/3 and 2 respectively.

6.2. Results for Deformed Mesh Models

We first show the effects of element shape quality improvement of the deformed mesh model and preservation of the mesh density and the shape approximation accuracy using some simple deformed mesh models. Then, we show the result of the application of our method to a simple mechanical part.

Figure 10 shows the initial mesh model (a) (the number of tetrahedra: 19,248, the number of vertices: 3,268), the deformed mesh model by changing a distance between planes (DP) (c), and the mesh model improved by our method (e) (the number of tetrahedra: 30,901, the number of vertices: 6,825). Figures 10 (b)(d)(f) show the cross section of (a)(c)(e). Table 2 shows the averages of stretches, minimum stretches, the averages of density errors, and the maximum density errors in the each mesh model. Figures 11 shows stretch histograms of each mesh model. These results show that our method could improve the quality of element shape of the deformed mesh models while preserving the mesh density of the initial mesh model in the deformed mesh model.

Figure 12 shows the initial mesh model (a) (the number of tetrahedra: 6,138, the number of vertices: 1,831), the deformed mesh model changing a radius of a cylinder (RC) (c), and the mesh model improved by our method (e) (the number of tetrahedra: 6,598, the number of vertices: 1,947). Figures 12 (b)(d)(f) show the stretch histograms of (a)(c)(e). Table 3 shows the averages of stretches, the minimum stretches, the averages of approximation errors, and the maximum approximation errors in the each mesh model. These results show that our method could provide a deformed mesh model which has approximation accuracy similar to that of the initial mesh model.

Figure 13 shows the initial mesh model (a) (the number of tetrahedra: 30,977, the number of vertices: 7,622), the deformed mesh model by changing DP, RC,

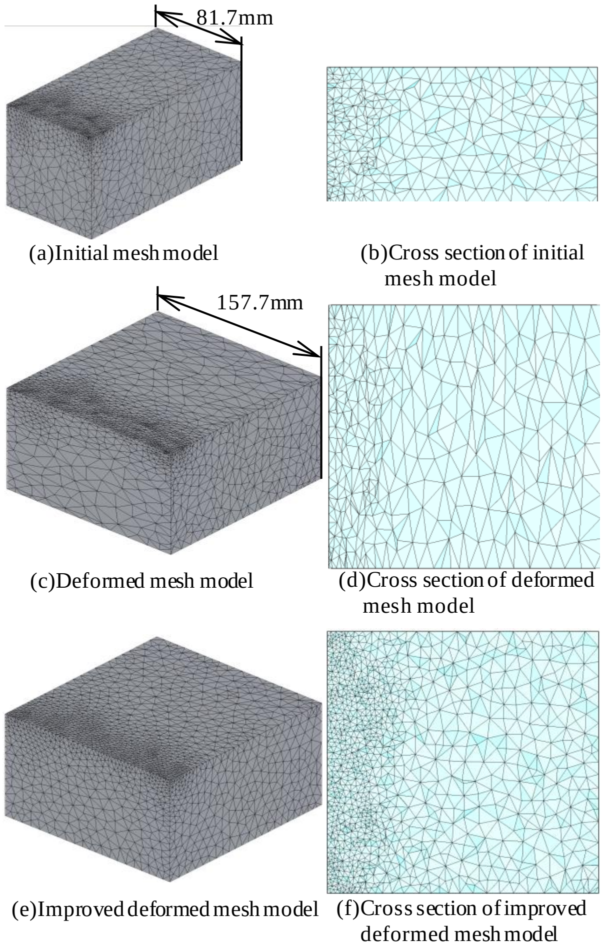


Figure 10 Result of improvement of deformed mesh model (changing DP)

Table 2 Mesh quality of each mesh model

Mesh Model	Stretch		Mesh Density Error	
	Ave	Min	Ave	Max
Initial Mesh Model	0.625	0.121	-	-
Deformed Mesh Model	0.511	0.078	0.289	4.076
Improved Deformed Mesh Model	0.694	0.158	0.205	2.424

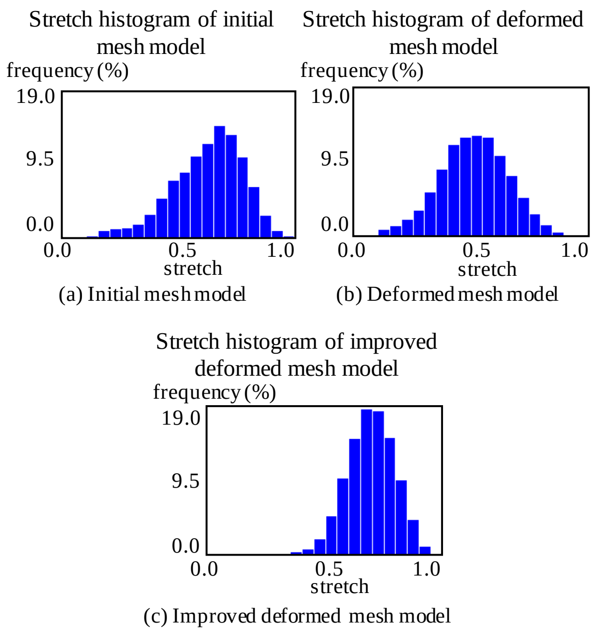
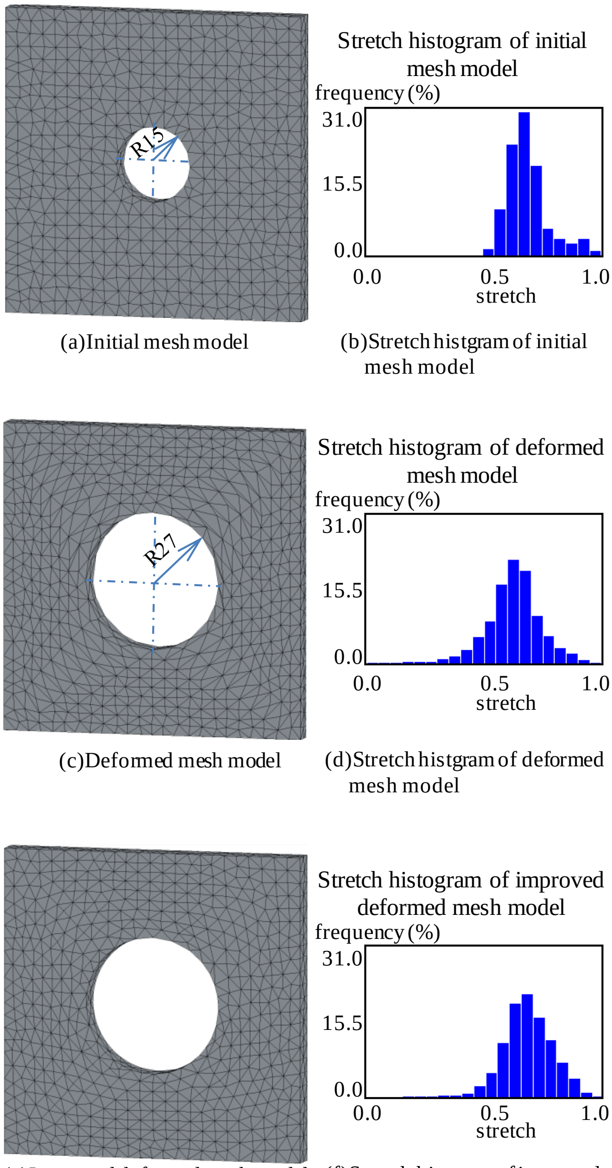


Figure 11 Stretch histogram



(a)Initial mesh model (b)Stretch histgram of initial mesh model

(c)Deformed mesh model (d)Stretch histgram of deformed mesh model

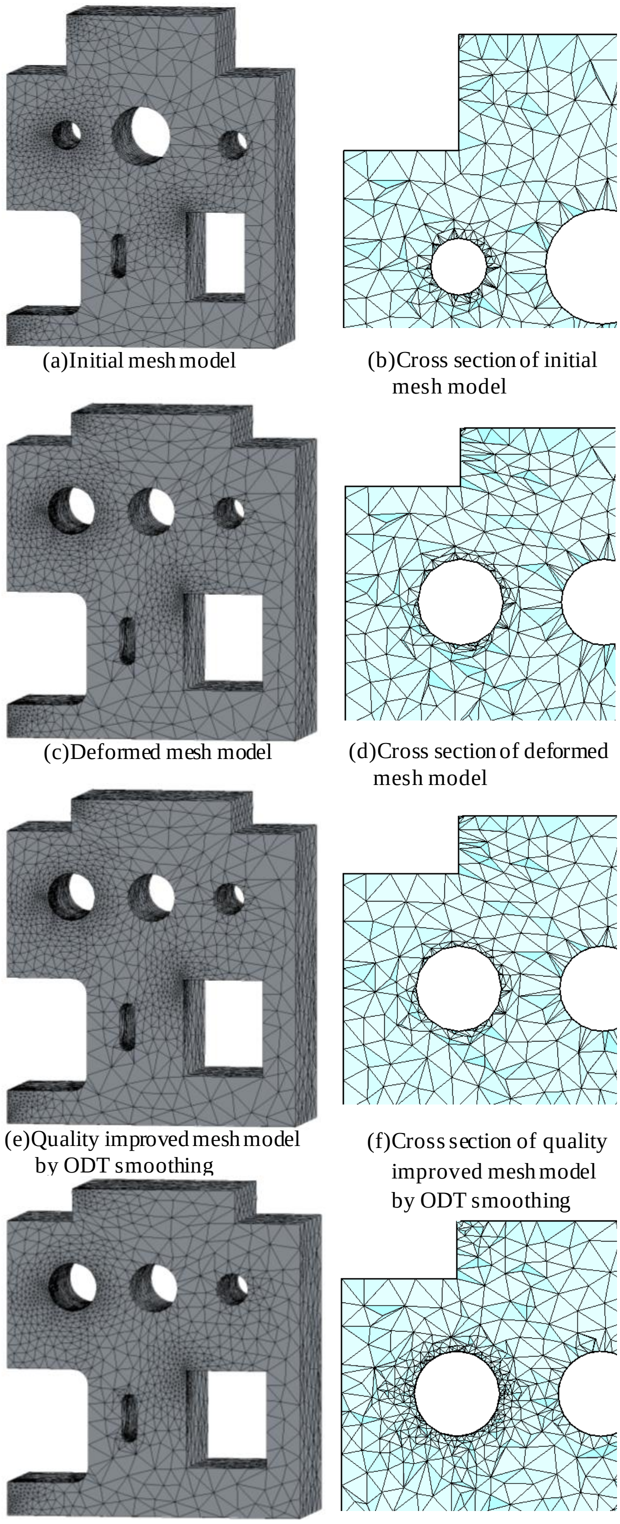
(e)Improved deformed mesh model (f)Stretch histgram of improved deformed mesh model

Figure 12 Result of improvement of deformed mesh models (changing RC)

Table 3 Mesh quality of each mesh model

Mesh Model	Stretch		Approximation Error	
	Ave	Min	Ave	Max
Initial Mesh Model	0.689	0.488	0.126	0.378
Deformed Mesh Model	0.625	0.015	0.226	0.680
Improved Deformed Mesh Model	0.679	0.172	0.075	0.287

and the position of a hole (PH) (c), the quality improved mesh model by ODT smoothing (e), and the quality improved mesh model by our method (g) (the number of tetrahedra: 43,505, the number of vertices: 10,257). Figures 13 (b)(d)(f)(h) show the cross section of (a)(c)(e)(g). Table 4 shows the averages of stretches, the minimum stretches, the averages of density errors, the maximum density errors, the averages of approximation errors, and the maximum approximation errors in the each mesh model. Figure 14 shows the stretch histograms of each mesh model. These results show that element shape qualities are improved while preserving



(a)Initial mesh model (b)Cross section of initial mesh model

(c)Deformed mesh model (d)Cross section of deformed mesh model

(e)Quality improved mesh model by ODT smoothing (f)Cross section of quality improved mesh model by ODT smoothing

(g)Quality improved mesh model by Our method (h)Cross section of quality improved mesh model by our method

Figure 13 Result of improvement of deformed mesh models (changing DP, RC and PH)

the mesh density and approximation accuracy of the initial mesh model in the deformed mesh model. The calculation time of the mesh quality improvement was 40[sec] using a standard PC (CPU: Core i7 3.40GHz, RAM: 8.00GB). In comparison with the quality improved mesh model by ODT smoothing(Fig. 13(g)),

Table 4 Mesh quality of each mesh model

Mesh Model	Stretch		Density Error*		Approximation Error	
	Ave	Min	Ave	Max	Ave	Max
Initial Mesh Model	0.549	0.068	-	-	0.072	0.139
Deformed Mesh Model	0.531	0.012	0.300	15.03	0.083	0.160
Quality Improved Mesh Model by ODT Smoothing	0.578	0.060	0.287	15.03	0.083	0.160
Quality Improved Mesh Model by Our Method	0.586	0.002	0.260	23.68	0.052	0.144

*The error is calculated for the edges in the corectly segmentatied region

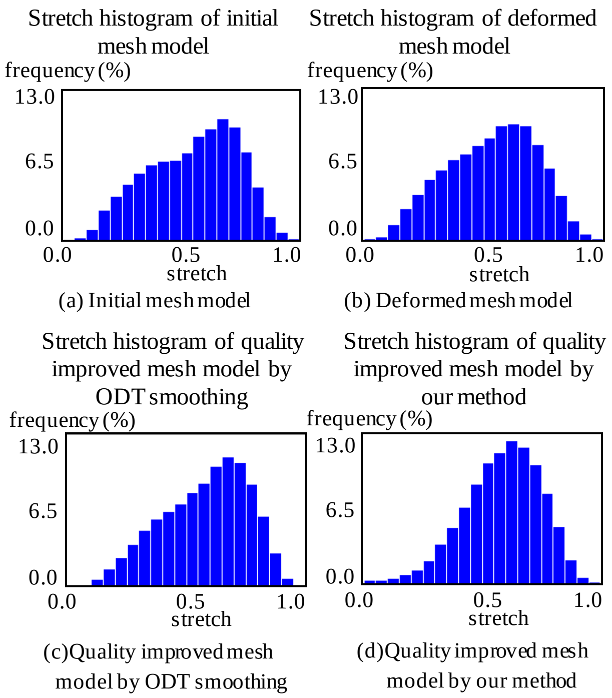


Figure 14 Stretch histogram

the quality improved mesh model by our method has better for the averages of stretches, density errors, and approximation errors. In addition, our method can improve maximum approximation error, too. However, for the minimum stretch and maximum density error, resulting mesh of our method is worse than one of ODT smoothing. Therefore, we will improve the minimum stretch in future work.

In large deformations, some inverted elements and slivers occurred (referred in Takano et al. [1]). In order to modify them, changing mesh connectivity is often needed. However, flipping operations used in our method could not modify them easily. Therefore we will add some operations, such as local remeshing, to modify them in future work.

7. Conclusion

In this paper, we proposed a quality improvement method for deformed tetrahedral mesh models. In our method, in order to improve all elements of deformed mesh models, ODT smoothing is applied to deformed mesh models in three phases: sharp mesh edge improvement, surface triangle improvement, and tetrahedron improvement. In each phase, we apply insertion and removal of vertices based on the target mesh density field and tolerances of the initial mesh model in order to preserve the mesh density and shape approximation accuracy of the initial mesh model in the deformed mesh model. In our method, ODT smoothing, insertion and removal of vertices are iteratively done.

The results of mesh improvement were evaluated by applying our method to some deformed models. The first example showed that our method can preserve the mesh density of the initial mesh model in the deformed mesh model, in addition to quality improvement of element shapes. The second example showed that our method can preserve the shape approximation accuracy of the initial mesh model in the deformed mesh model. Similar improvement effects could be confirmed for the simple mechanical part.

Our future work will focus on extending our method for improving mesh models, including free-form surfaces, and developing a new method for modifying inverted elements and slivers. In addition, we will compare results of our method with ones of other smoothing methods or remeshing methods.

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